1. UNIFORMLY ACCELERATED MOTION

When a body moves with uniform acceleration, the velocity v and the distance traveled s increase during time t. In this case, the velocity increases with increasing distance.

The instantaneous velocity through the time interval t is determined by the equation:

$$v(t) = a \cdot t \tag{1.1}$$

The distance traveled is determined by the equation

$$s(t) = \frac{1}{2} \cdot a \cdot t^2 \tag{1.2}$$

From where it follows:

$$v(s) = \sqrt{2 \cdot a \cdot s} \tag{1.3}$$

and

$$v^2(s) = 2 \cdot a \cdot s \tag{1.4}$$

The instantaneous velocity can be determined using the equation:

$$v = \frac{\Delta s}{\Delta t} \tag{1.5}$$

To measure the instantaneous velocity, a flag of the interrupter of known width Δs is attached to the cart and interrupts the beam of the photoelectric sensor when the cart passes by it. The break time of the beam Δt is measured using a digital counter.

If we plot the squares of the instantaneous velocity for each run calculated from the moments of the beam break, depending on the distances traveled, we can expect that in the case of uniform acceleration there will be a linear dependence described by Equation 1.4. The gradient of the line passing through the starting point is equal to twice the acceleration (Fig.1.1).

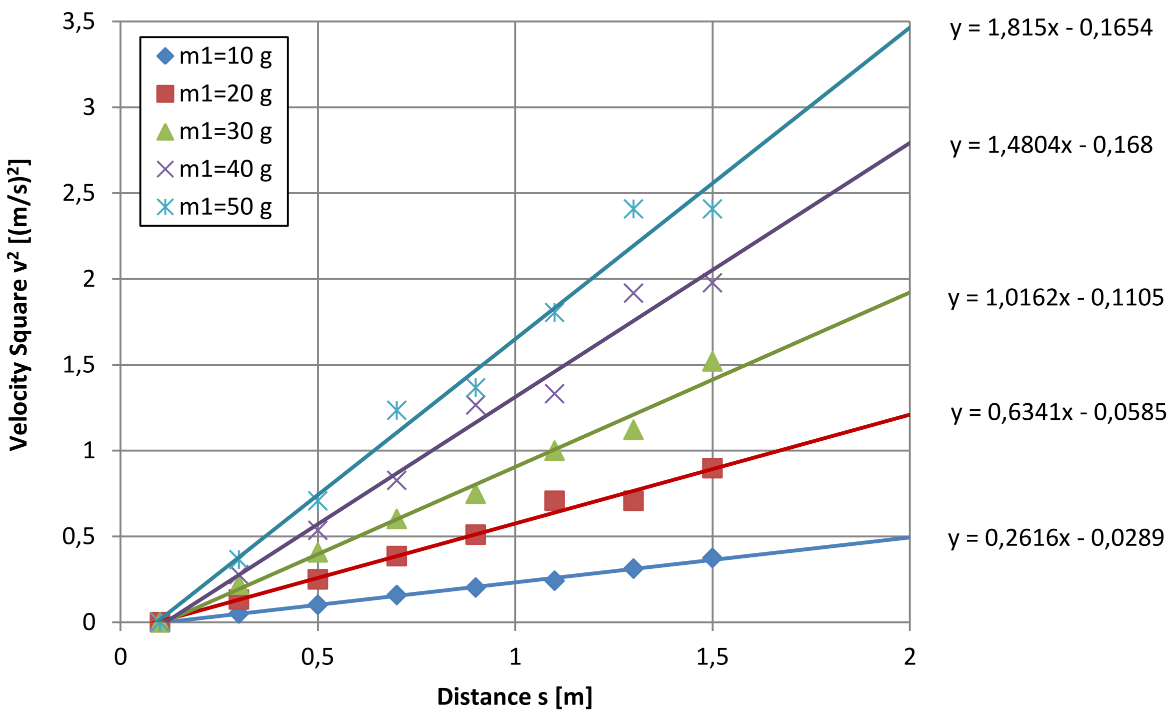

Fig. 1.1 – Velocity square-Distance (v^2-s) plot for m_2=500 g and for different values of m_1

2. MOTION WITH UNIFORM ACCELERATION

In this laboratory work, constant acceleration is the result of the action of a constant accelerating force F:

$$a = F/m \qquad (2.1)$$

The trolley accelerates evenly because it is pulled by the rope, which is acted upon by a constant force created by a weight of known mass attached to the free end of the rope. The rope is thrown over a pulley with spokes, which, when rotating, periodically interrupt the photoelectric sensor.

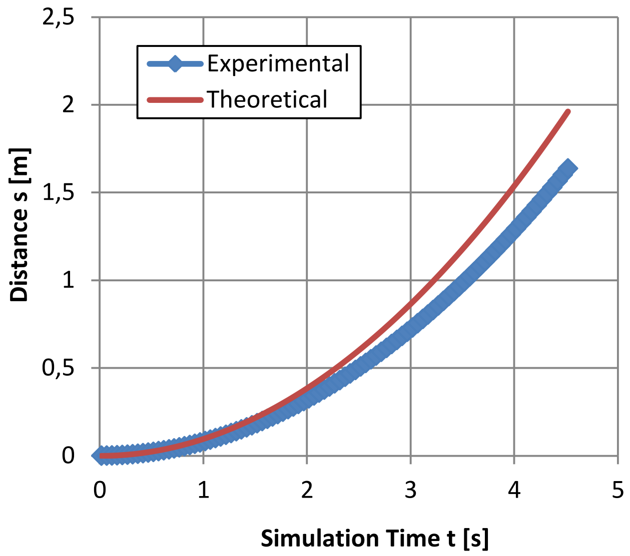

Fig. 2.1 – Dependence of the distance traveled on time

Sensor data is read and processed automatically. In this case, during the rotation of the pulley, the time t is measured at the moment when the spokes interrupt the light beam. During the experiment, the dependences of distance s (Fig.2.1), velocity v (Fig.2.2) and acceleration a (Fig.2.3) on the time t of movement of the trolley are built on the screen.

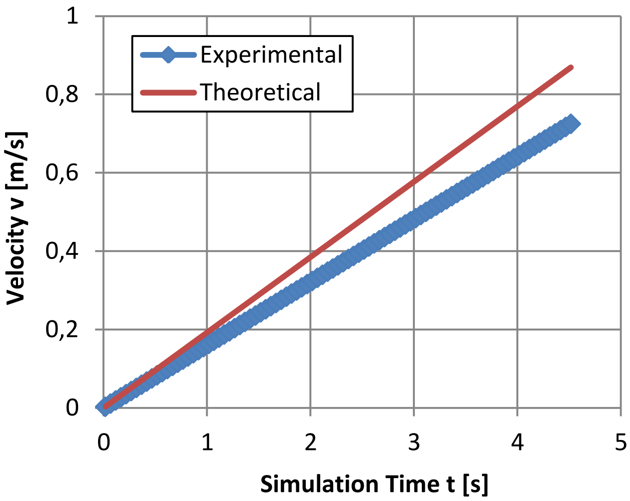

Fig. 2.2 – Dependence of the velocity on time

Let m_1 the mass of the weight hanging from the rope and m_2 be the mass of the trolley. Since the mass m_1 is also subject to acceleration, the following values should be used in equation (2.1):

$$m = m_1 + m_2 \qquad (2.2)$$

$$F = m_1 \cdot g \qquad (2.3)$$

Therefore

$$a = \frac{m_1}{m_1+m_2} \cdot g \qquad (2.4)$$

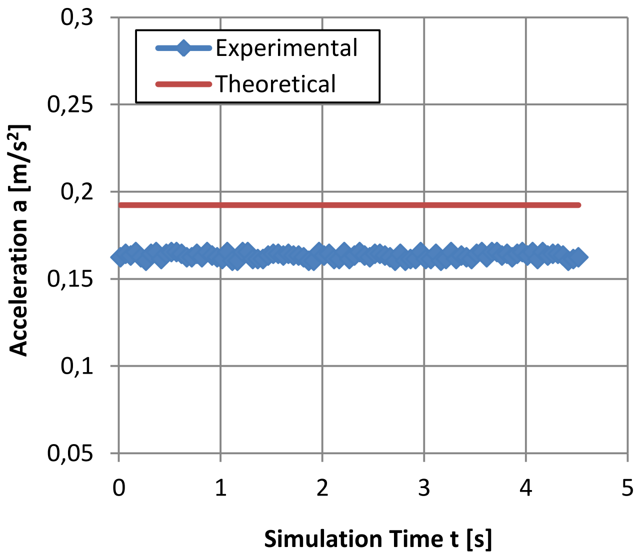

Fig. 2.3 – Dependence of the acceleration on time

3. LAWS OF COLLISIONS

Theoretically, in a reference frame with a common center of gravity, the total momentum of two bodies of mass m_1 and m_2 is zero both before and after the collision.

$$p_1 + p_2 = p_1{}' + p_2{}' = 0 \tag{3.1}$$

where p_1, p_2 – individual momentums of the bodies before collision; p_1', p_2' – individual momentums of the bodies after collision.

The kinetic energy of two colliding bodies in one frame of reference is determined by the equation

$$E = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} \tag{3.2}$$

When all the kinetic energy is stored in the frame of reference of the common center of gravity, we speak of elastic collisions. In an inelastic collision, all of the energy transforms into another form.

Using a slide track as a frame of reference, momentum conservation is described by the expression:

$$p_1 + p_2 = p_1' + p_2' = p = const \tag{3.3}$$

As a result of conservation of momentum, the velocity of the gravity center is determined by

$$v_c = \frac{p}{m_1+m_2} \tag{3.4}$$

and the kinetic energy of the gravity center is

$$E_c = \frac{m_1+m_2}{2} \cdot v_c^2 \tag{3.5}$$

These equations are valid for both elastic and inelastic collisions. In this experiment, the second body is initially at rest before the collision. Consequently, the momentum conservation law (Equation 3.3) is described by the expression

$$p = m_1 \cdot v_1 = m_1 \cdot v_1{}' + m_2 \cdot v_2{}' \tag{3.6}$$

where v_1' and v_2' have different values after elastic collision, but are the same after inelastic collision.

In an elastic collision, the flat plate on the first body collides with the stretched elastic band on the second body. In an inelastic collision, the long pointed spike of the first body is pressed into the clay on the second body.

The masses of the colliding bodies can be changed by adding additional weights.

For elastic collision, the following equations apply:

$$p_1' = \frac{m_1-m_2}{m_1+m_2} \cdot p; \quad p_2' = \frac{2 \cdot m_2}{m_1+m_2} \cdot p \tag{3.7}$$

$$E = \frac{m_1}{2} \cdot v_1^2 = \frac{m_1}{2} \cdot v_1^{2\prime} + \frac{m_2}{2} \cdot v_2^{2\prime} \tag{3.8}$$

If the collision is an inelastic, only the kinetic energy of the gravity center is conserved. The kinetic energy of the gravity center can be calculated using equations (3.4), (3.5) and (3.6):

$$E_c = \frac{m_1}{m_1+m_2} \cdot \frac{m_1}{2} \cdot v_1^2 = \frac{m_1}{m_1+m_2} \cdot E \tag{3.9}$$

The velocity of each body can be determined by knowing the width of the chopper flag ds and the time dt recorded by the digital counter

$$v = \frac{ds}{dt} \tag{3.10}$$

4. FREE FALL

In this experiment, a steel ball is attached to the special mechanism at a predetermined height. As soon as the ball is released into free fall, the digital time counter starts.

After the ball has fallen from height h, it hits the target on the lower platform, which stops the measurement of time at time t.

The initial velocity of the ball at time $t_0=0$ is equal to zero $v_0=0$. Therefore, the distance traveled in time t is determined as follows

$$s = \frac{1}{2} \cdot g \cdot t^2 \tag{4.1}$$

Time measurements for different heights of fall should be plotted on a graph of distance s versus time (Fig.4.1). Distance s is not linearly proportional to time t. To get a straight line, the distance must be related to the square of the time (Fig.4.2). The rectilinear dependence found in this way confirms equation (4.1). The slope of such a line corresponds to the acceleration of gravity g.

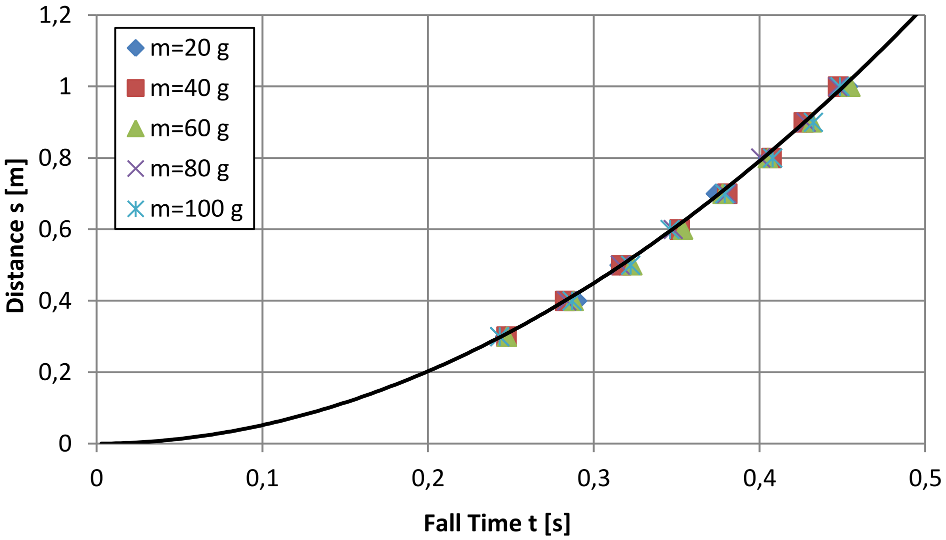

Fig. 4.1 – Distance-Time (s-t) plot

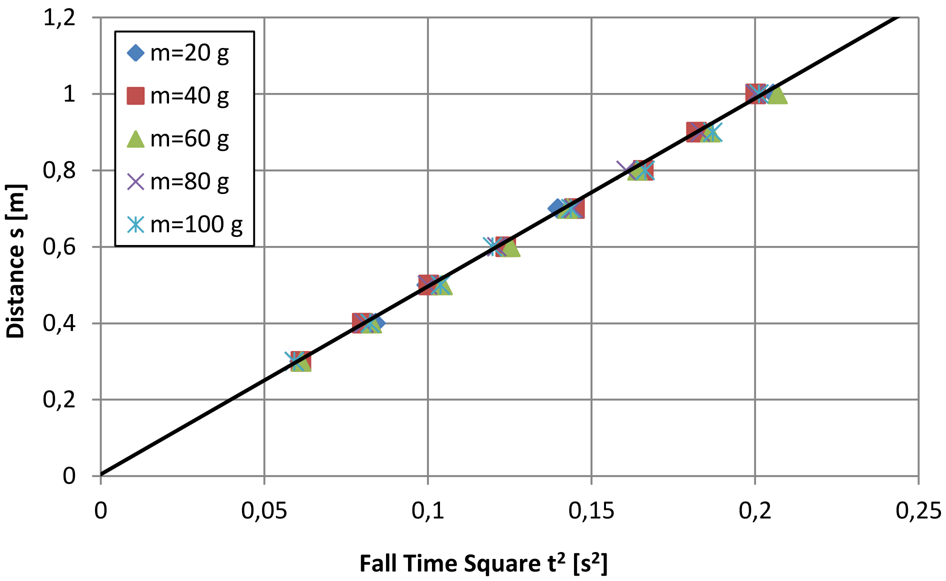

Fig. 4.2 – Distance-Time square (s-t^2) plot

5. INCLINED LAUNCH

When calculating the theoretical flight curve of a ball thrown at an angle to the horizon, for simplicity, we take the center of the ball as the origin of the coordinate system and neglect air friction. Thus, the ball maintains its initial velocity in the horizontal direction

$$v_x(0) = v_0 \cdot \cos(\alpha) \tag{5.1}$$

In this case, at time t, the horizontal distance traveled is equal to

$$x(t) = v_0 \cdot \cos(\alpha) \cdot t \tag{5.2}$$

In the vertical direction, under the action of gravity, the ball experiences the acceleration of gravity g. Consequently, at time t, its vertical velocity component is

$$v_y(t) = v_0 \cdot \sin(\alpha) - g \cdot t \tag{5.3}$$

and the vertical distance component is

$$y(t) = v_0 \cdot \sin(\alpha) \cdot t - \frac{1}{2} \cdot g \cdot t^2 \tag{5.4}$$

The ball flight curve has the shape of a parabola (Fig.5.1), since it corresponds to the equation

$$y(x) = \tan(\alpha) \cdot x - \frac{1}{2} \cdot \frac{g}{(v_0 \cdot \cos(\alpha))^2} \cdot x^2 \tag{5.5}$$

The ball reaches the highest point of the parabola at time t_1

$$t_1 = \frac{v_0 \cdot \sin(\alpha)}{g} \tag{5.6}$$

At time t_2, the ball is again at the initial height h=0

$$t_2 = 2 \cdot \frac{v_0 \cdot \sin(\alpha)}{g} \tag{5.7}$$

The height of the parabola can be defined as

$$h = y(t_1) = \frac{v_0^2}{2 \cdot g} \cdot \sin^2(\alpha) \tag{5.8}$$

The width of the parabola can be defined as

$$s = x(t_2) = 2 \cdot \frac{v_0^2}{g} \cdot \sin(\alpha) \cdot \cos(\alpha) \tag{5.9}$$

The maximum flight curve width s_{max} is reached when the launch angle α=45°. Starting at this maximum width, the initial ball velocity can be calculated using Equation 5.9

$$v_0 = \sqrt{g \cdot s_{max}} \tag{5.10}$$

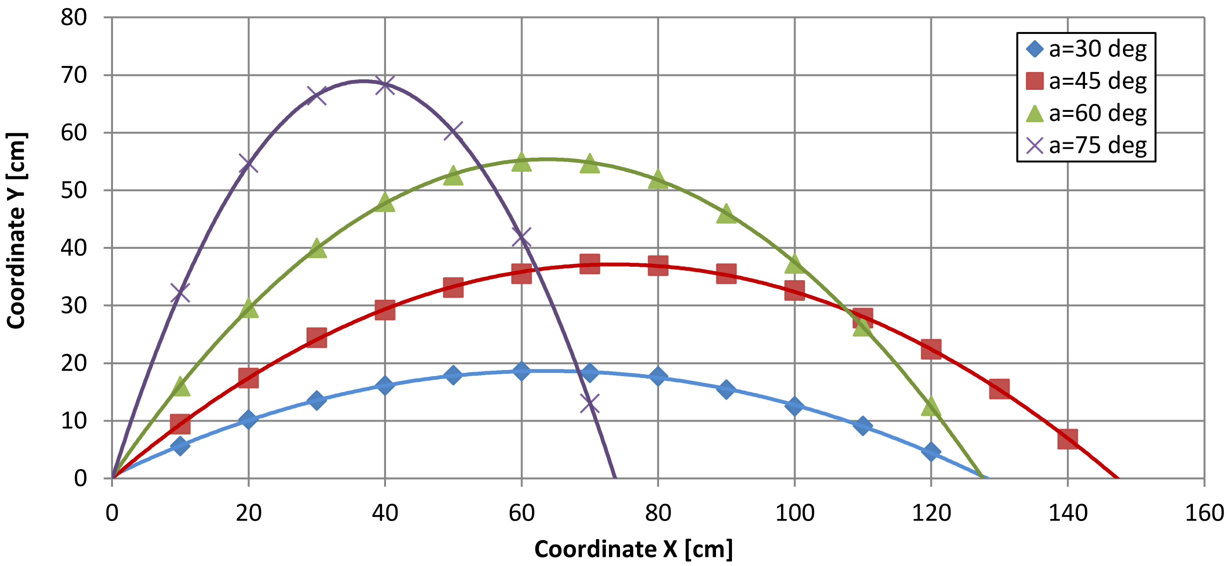

Fig. 5.1 – Flight trajectories of a ball released at different angles to the horizon

6. PRECESSION AND NUTATION OF A GYROSCOPE

The large disc of the gyroscope rotates around an axis fixed at the fulcrum with the low friction. The counterweight is adjusted so that the fulcrum is aligned with the gyro's center of gravity. If the gyroscope is in equilibrium and the disk rotates, the moment L will be constant:

$$L = I \cdot \omega_R \tag{6.1}$$

where I – moment of inertia and ω_R – angular velocity of the disk.

The moment of inertia of the gyroscope disk is determined by the expression:

$$I = \frac{1}{2} \cdot M \cdot R^2 \tag{6.2}$$

where M – disc mass and R – disc radius.

If an additional weight is placed on the axis of rotation by adding mass m, this produces a torque τ that changes the angular momentum of the gyroscope

$$\tau = m \cdot g \cdot r = \frac{dL}{dt} \tag{6.3}$$

where r is the distance from the fulcrum of the axis of rotation to the place where the additional weight is applied.

In this case, the axis of rotation moves to the next angle

$$d\varphi = \frac{dL}{L} = \frac{m \cdot g \cdot r \cdot dt}{L} \tag{6.4}$$

As a result of the addition of weight, the precession of the gyroscope begins. Then the angular velocity of the precessional motion can be determined as

$$\omega_P = \frac{d\varphi}{dt} = \frac{m \cdot g \cdot r}{L} = \frac{m \cdot g \cdot r}{I \cdot \omega_R} \tag{6.5}$$

where $\omega = 2\pi/T = 2\pi \cdot f$

$$\frac{1}{T_R} = f_R = \frac{m \cdot g \cdot r}{I} \cdot T_P \tag{6.6}$$

If the disc is set to rotate without additional external torque and the axis of rotation is slightly tilted to one side, the gyroscope will indicate nutation. Then the angular velocity of nutation is directly proportional to the angular velocity of rotation:

$$\omega_N = C \cdot \omega_R \quad \text{and} \quad T_R = C \cdot \omega_N \quad \text{where } C = const \tag{6.7}$$

The periods of rotation, precession, and nutation are determined from the experimental data recorded by photoelectric sensors automatically during the experiment. According to Equation 6.6, the period of precession is inversely proportional to the period of rotation, while Equation 6.7 shows that the period of nutation is directly proportional to the period of rotation. In the corresponding graphs, the measured values will lie along a straight line through the origin (Fig. 6.1 and Fig. 6.2).

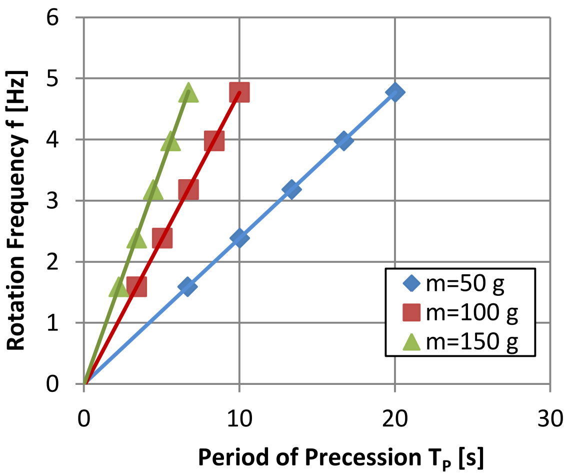

Fig. 6.1 – Frequence-Period of precession (f-T_P) plot

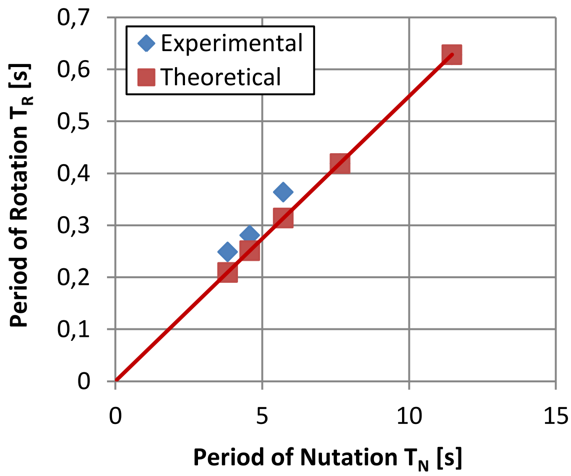

Fig. 6.2 – Period of rotation-Period of nutation (T_R-T_N) plot

7. ROTATIONAL MOTION WITH UNIFORM ACCELERATION

The relationship between the torque M, which is applied to a rigid body, the moment of inertia J maintained so that it can rotate, and the angular acceleration ε is:

$$M = J \cdot \varepsilon \tag{7.1}$$

If the applied torque is constant, the body rotates with constant angular acceleration.

In this experiment, motion with low friction of the disk begins at time t_0=0 with zero initial angular velocity ω=0, and in time t it rotates through an angle

$$\varphi = \frac{1}{2} \cdot \varepsilon \cdot t^2 \tag{7.2}$$

The torque M is created by the weight of the accelerating mass m_M acting at a distance r_M from the axis of rotation of the body, and is

$$M = r_M \cdot m_M \cdot g \tag{7.3}$$

If two additional weights of mass m_J are fixed on the horizontal rod of the rotating system at the same fixed distance r_J from the axis of rotation, the moment of inertia increases to

$$J = J_0 + 2 \cdot m_J \cdot r_J^2 \tag{7.4}$$

where J_0 – moment of inertia of rotating system without additional weights.

To determine the angular acceleration ε as a function of the variables M and J, it is necessary to measure the time t (90°) required to rotate the disc through an angle of 90° with different values of the variable in both cases. In this particular case, the angular acceleration is

$$\varepsilon = \frac{\pi}{t(90^0)^2} \tag{7.5}$$

Fig.7.1 and Fig.7.2 illustrate the dependence of the disk rotation angle on time, and Fig.7.3 shows the dependence of the angular acceleration on the applied torque.

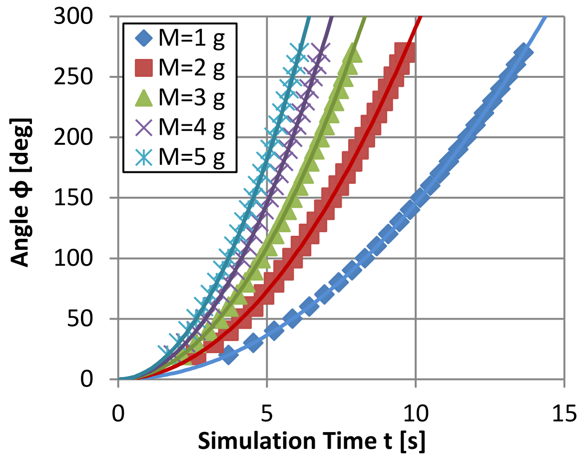

Fig. 7.1 – Angle-Time (ϕ-t) plot

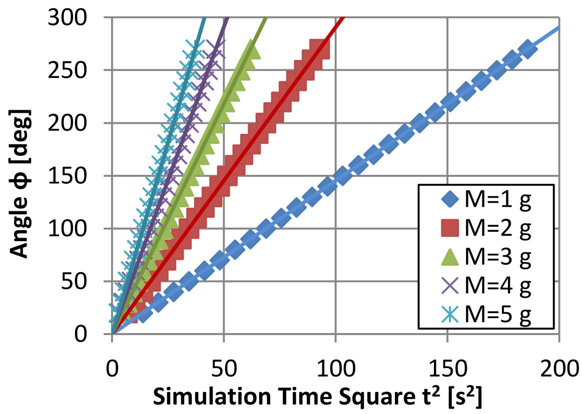

Fig. 7.2 – Angle-Time Square (ϕ-t^2) plot

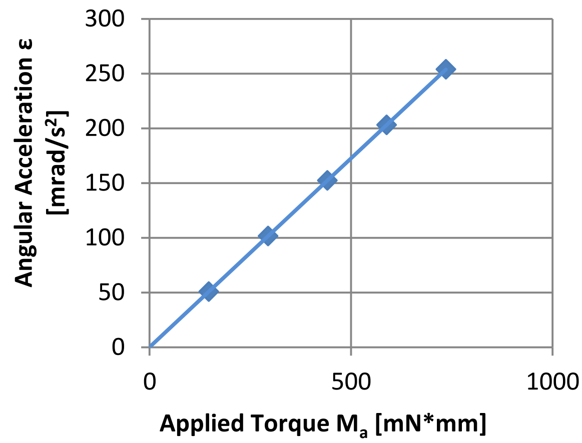

Fig. 7.3 – Angular acceleration-Applied torque (ε-M) plot

8. MOMENT OF INERTIA OF A HORIZONTAL ROD

In this experiment, a rotating disk with an attached horizontal rod is considered, to which two additional weights of mass m are symmetrically attached at a distance r from the axis of rotation. For this system, the moment of inertia is:

$$J = J_0 + 2 \cdot m \cdot r^2 \tag{8.1}$$

where J_0 is the moment of inertia of the disc and the rod without additional weights.

Since the rotating disk is elastically connected by a helical spring to a rigid support, the moment of inertia can be determined from the period of torsional oscillations of the disk relative to its rest position:

$$T = 2 \cdot \pi \cdot \sqrt{\frac{J}{D_r}} \tag{8.2}$$

where D_r is the torsional coefficient of the coil spring.

Equation 8.2 shows that the greater the moment of inertia J of the disk with a horizontal rod attached to it, depending on the mass m and the distance r, the longer the period of oscillation T.

During the experiment, the periods of torsional vibrations of the disk are recorded using a digital counter. In this case, the following equation is derived from equation 8.2 to determine the moment of inertia:

$$J = D_r \cdot \frac{T^2}{4\pi^2} \tag{8.3}$$

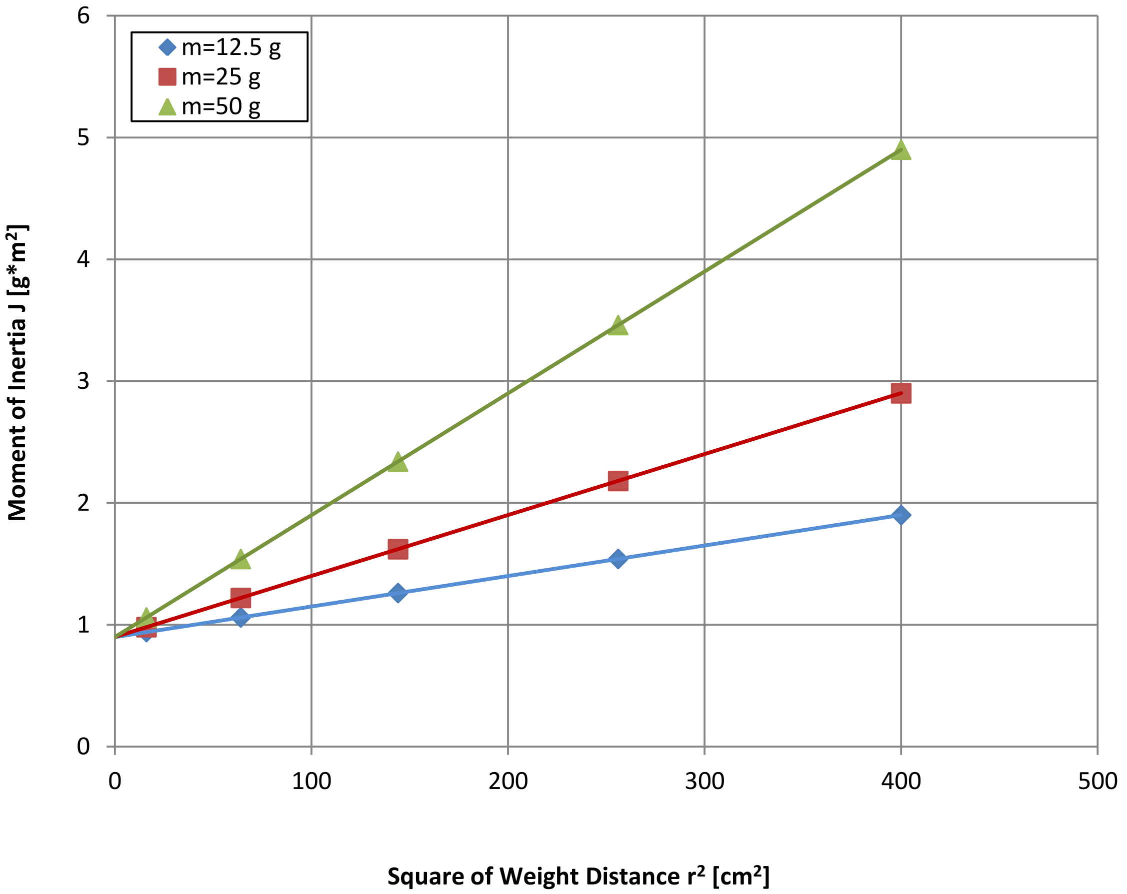

Fig. 8.1 – Moment of inertia J of rotating disc with horizontal rod as a function of the square of the distance r from the axis of rotation for different additional weights of mass m

9. MOMENT OF INERTIA OF VARIOUS TEST BODIES

The moment of inertia is defined as an integral over the volume:

$$J = \int_v r_s^2 \cdot \rho(r) \cdot dV \qquad (9.1)$$

where r_s – component perpendicular to the axis of rotation; ρ(r) – mass distribution in the body.

One of the elements of this laboratory work is a horizontal rod, fixed on a hinge, connected to a spiral spring. Two weights of mass m are placed on the rod symmetrically at a distance r from the axis of rotation. For this rotating system, the moment of inertia is defined as

$$J = J_0 + 2 \cdot m \cdot r^2 \qquad (9.2)$$

where J_0 is the moment of inertia of the rod without additional weights.

In the course of the experiment, various bodies of the correct geometric shape are installed to the axis of rotation above the rod so that they can oscillate. If the period of oscillation of the system is equal to T, then the following is true:

$$T = 2 \cdot \pi \cdot \sqrt{\frac{J}{D_r}} \qquad (9.3)$$

where D_r is the spring torsional coefficient.

This means that the oscillation period T increases with the moment of inertia J. The torsional coefficient of a spiral spring can be determined using a laboratory dynamometer:

$$D_r = \frac{F \cdot r}{\alpha} \qquad (9.4)$$

where F – force applied to the spring to rotate the rod through an angle α.

Fig.9.1 shows the dependence of the force F, recorded on the dynamometer scale, on the reverse arm 1/r of the application of force relative to the axis of rotation. Fig.9.2 shows the dependence of the moment of inertia of additional weights J_m on a horizontal rod from the square of their distance r^2 to the axis of rotation. Fig.9.3 illustrates the verification of the Huygens-Steiner theorem when examining the moment of inertia of a thin disc with holes.

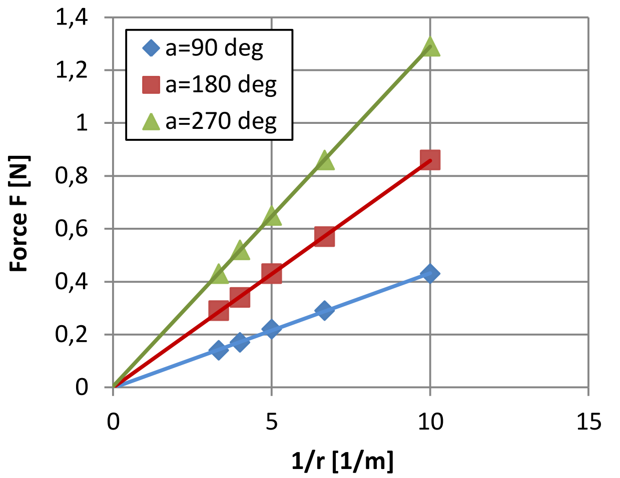

Fig. 9.1 – Applied force-Reverse arm (F-1/r) plot

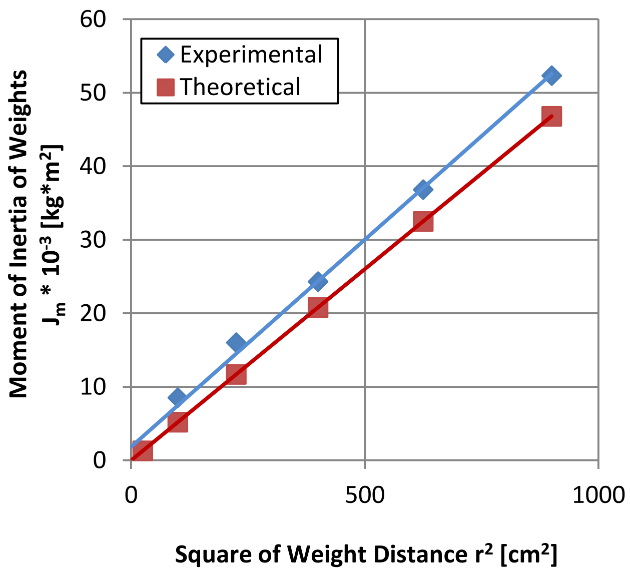

Fig. 9.2 – Moment of Inertia-Distance square (J_m-r^2) plot

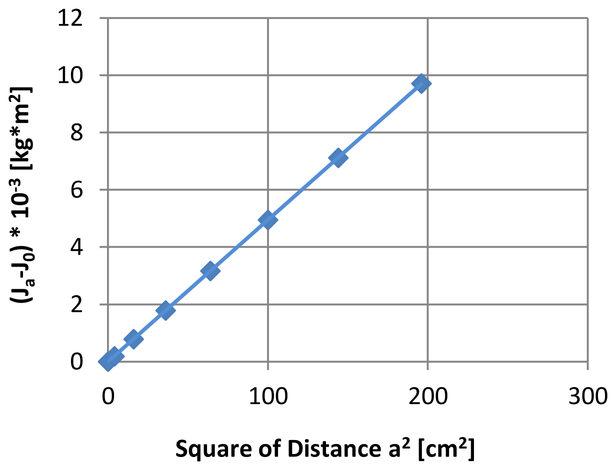

Fig. 9.3 – Verification of the Huygens-Steiner theorem for thin disc with holes

10. MAXWELL'S WHEEL

When rolling up and down, Maxwell's wheel moves at a velocity v. This velocity obeys the following fixed dependence on the angular velocity ω, with which the wheel rotates around its axis:

$$v = \omega \cdot r \qquad (10.1)$$

where r – radius of wheel axis.

In this case, the total energy is

$$E = m \cdot g \cdot h + \frac{1}{2} \cdot J \cdot \omega^2 + \frac{1}{2} \cdot m \cdot v^2 =$$
$$= m \cdot g \cdot h + \frac{1}{2} \cdot m \cdot \left(\frac{J}{m \cdot r^2} + 1\right) \cdot v^2 \qquad (10.2)$$

where m is the wheel mass, J is the moment of inertia, h is the height above the bottom point, g is the acceleration of gravity.

When the wheel moves downward, the acceleration is defined as

$$a = g / \left(\frac{J}{m \cdot r^2} + 1\right) \qquad (10.3)$$

This acceleration is determined in the experiment by the distance traveled s in time t (Fig.10.1):

$$s = \frac{1}{2} \cdot a \cdot t^2 \qquad (10.4)$$

It can also be determined by the instantaneous velocity achieved in time t (Fig.10.2):

$$v = a \cdot t \qquad (10.5)$$

From equation 10.3, the following is true

$$J = m \cdot r^2 \cdot \left(\frac{g}{a} - 1\right) \qquad 10.6)$$

The kinetic energy E_{kin} is calculated as

$$E_{kin} = \frac{1}{2} \cdot m \cdot \left(\frac{J}{m \cdot r^2} + 1\right) \cdot v^2 \qquad (10.7)$$

The potential energy is determined by the equation

$$E_{pot} = m \cdot g \cdot h \qquad (10.8)$$

Energy losses are due to the fact that the friction force acts against the direction of movement, and there is also a change in the direction of movement at the lowest point (Fig.10.3).

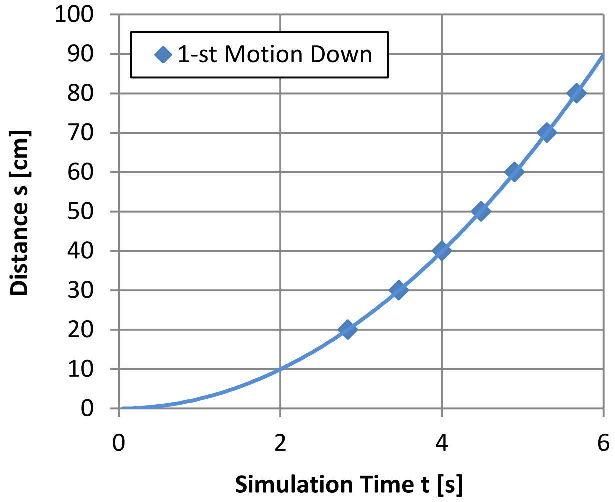

Fig. 10.1 – Distance-Time (s-t) plot for first motion down

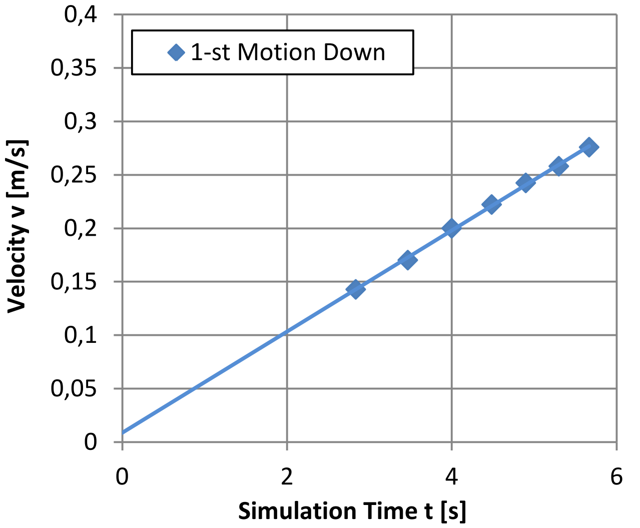

Fig. 10.2 – Velocity-Time (v-t) plot for first motion down

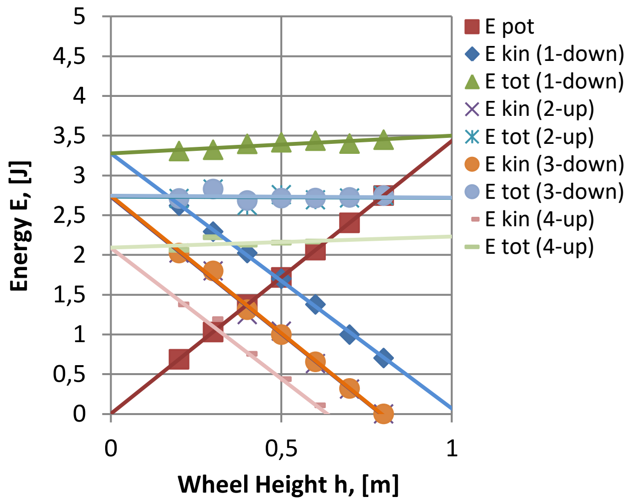

Fig. 10.3 – Energy-Height (E-h) plot for the initial four wheel motions

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